Comment on "Superfluid Turbulence from Quantum Kelvin Wave to Classical Kolmogorov Cascades"

In a recent Letter [1], Yepez *et al.* performed numerical simulations of the Gross-Pitaevskii equation (GPE) using a novel unitary quantum algorithm with very high resolution. They claim to have found new power-law scalings for the incompressible kinetic energy spectrum: "... (the) solution clearly exhibits three power law regions for $E_{\rm kin}^{\rm incomp}(k)$: for small k the Kolmogorov $k^{-(5/3)}$ spectrum while for high k a Kelvin wave spectrum of k^{-3} ... ."

In this Comment we point out that the high wave number k^{-3} power law observed by Yepez *et al.* is an artifact stemming from the definition of the kinetic energy spectra and is thus not directly related to a Kelvin wave cascade. Furthermore, we clarify a confusion about the wave number intervals on which Kolmogorov and Kelvin wave cascades are expected to take place.

The dynamics of a superflow is described by the GPE

$$\partial_t \psi = ic/(\sqrt{2}\xi)(\psi - |\psi|^2\psi + \xi^2\nabla^2\psi), \qquad (1)$$

where the complex field ψ is related by Madelung's transformation $\psi = \sqrt{\rho}\exp(i\frac{\phi}{\sqrt{2}c\xi})$ to the density ρ and velocity $\vec{v} = \nabla\phi$ of the superfluid. In these formulas, ξ is the coherence length and c is the velocity of sound (for a fluid of unit mean density). The superflow is irrotational, except on the nodal lines $\psi = 0$, which are the superfluid vortices.

The GPE dynamics Eq. (1) conserves the energy that can be written as the sum (the space integral) of three parts: the kinetic energy $\mathcal{E}_{\rm kin} = 1/2(\sqrt{\rho}v_j)^2$, the internal energy $\mathcal{E}_{\rm int} = (c^2/2)(\rho - 1)^2$, and the quantum energy $\mathcal{E}_q = c^2\xi^2(\partial_j\sqrt{\rho})^2$. Using Parseval's theorem, one can define the corresponding energy spectra, e.g., the kinetic energy spectrum $E_{\rm kin}(k)$, as the sum over the angles of $|\frac{1}{(2\pi)^3} \times \int d^3r e^{ir_jk_j}\sqrt{\rho}v_j|^2$ [2].

The 3D angle-averaged spectrum of a smooth isolated vortex line is known to be proportional to that of the 2D axisymmetric vortex, an exact solution of Eq. (1) given by $\psi^{\rm vort}(r) = \sqrt{\rho(r)}\exp(\pm i\varphi)$ in polar coordinates (r, φ). The corresponding velocity field $v(r) = \sqrt{2}c\xi/r$ is azimuthal and the density profile, of characteristic spatial extent ξ, verifies $\sqrt{\rho(r)} \sim r$ as $r \to 0$ and $\sqrt{\rho(r)} = 1 + O(r^{-2})$ for $r \to \infty$. Thus $\sqrt{\rho}v_j$ has a small r singular behavior of the type r^0 and behaves as r^{-1} at large r. In general, for a function scaling as $g(r) \sim r^s$ the (2D) Fourier transform is $\hat{g}(k) \sim k^{-s-2}$ and the associated spectrum scales as k^{-2s-3}. Thus $E_{\rm kin}(k)$ scales as k^{-3} for $k \gg k_\xi \sim \xi^{-1}$ and as k^{-1} for $k \ll k_\xi$ [3].

Following the above discussion, the k^{-3} power law observed in [1] is an artifact stemming from the definition of the kinetic energy spectra and is not directly related to a Kelvin wave cascade.

Another very important scale, not discussed in the Letter [1], is the scale ℓ of the mean intervortex distance. The hydrodynamic (Kolmogorov) energy cascade is expected to end at $k_\ell \sim \ell^{-1}$ [2] and the Kelvin wave cascade to begin there, after an eventual bottleneck [4]. Note that $\ell_I \gg \ell \gg \xi$, where ℓ_I is the energy containing scale. We thus believe that nothing particularly interesting is taking place between k_ξ and the maximum wave number $k_{\max}$ of the simulation and that there is a confusion in [1] between k_ℓ and k_ξ.

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Giorgio Krstulovic and Marc Brachet
Laboratoire de Physique Statistique de l'Ecole Normale Supérieure
associé au CNRS et aux Universités Paris VI et VII
24 Rue Lhomond, 75231 Paris, France

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