Supplemental Material: Lack of self-similarity in transverse velocity increments and circulation statistics in two-dimensional turbulence

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RELATION BETWEEN CIRCULATION AND TRANSVERSE SCALING EXPONENTS

We consider the circulation $\Gamma_r(\boldsymbol{s}_0)$ around a squared loop of size r, with one corner of the loop placed at $\boldsymbol{s}_0 = (x_0, y_0)$. It follows that

$$\langle|\Gamma_r|^p\rangle = \frac{1}{V}\int |\Gamma_r(\boldsymbol{s}_0)|^p \mathrm{d}\boldsymbol{s}_0 \leq \frac{1}{V}\int \left[\int_{x_0}^{x_0+r} |u_x(x, y_0+r) - u_x(x, y_0)|\mathrm{d}x + \int_{y_0}^{y_0+r} |u_y(x_0+r, y) - u_y(x_0, y)|\mathrm{d}y\right]^p \mathrm{d}\boldsymbol{s}_0, \tag{1}$$

where we applied the triangular inequality several times. Following a similar procedure to the one in Iyer et al. [1], we now apply the Hölder inequality for each of the two integrals in the right hand side. It leads to

$$\langle|\Gamma_r|^p\rangle \leq \frac{1}{V}\int \left[r^{1/q}\left(\int_{x_0}^{x_0+r} |u_x(x, y_0+r) - u_x(x, y_0)|^p \mathrm{d}x\right)^{1/p} + r^{1/q}\left(\int_{y_0}^{y_0+r} |u_y(x_0+r, y) - u_y(x_0, y)|^p \mathrm{d}y\right)^{1/p}\right]^p \mathrm{d}\boldsymbol{s}_0 \tag{2}$$

with p and q satisfying $p^{-1} + q^{-1} = 1$ for $p, q > 1$.

For a sufficiently large Reynolds numbers, assuming homogeneity, isotropy, and at a fixed r in the inertial range, we can approximate each inner integral by $r\langle|\delta u_r^\perp|^p\rangle = rS_p^\perp(r)$. Fig. 1 shows the validity of this approximation in the inertial range for RUN-A. The outer integral cancels out and we obtain

$$\langle|\Gamma_r|^p\rangle \leq 2^p r^{p/q} r^{p/p} (S_p^\perp)^{p/p} = 2^p r^p S_p^\perp(r). \tag{3}$$

Finally, we use the fact that the circulation moments and TSFs follow the scaling properties $\langle|\Gamma_r|^p\rangle \sim (r/L_f)^{\lambda_p}$ and $S_p^\perp \sim (r/L_f)^{\zeta_p^\perp}$, with L_f the forcing scale. For the inertial range of the inverse energy cascade in two-dimensional turbulence, we take the limit $r/L_f \gg 1$, so we obtain an inequality for the scaling exponents

$$\lambda_p \leq \zeta_p^\perp + p. \tag{4}$$

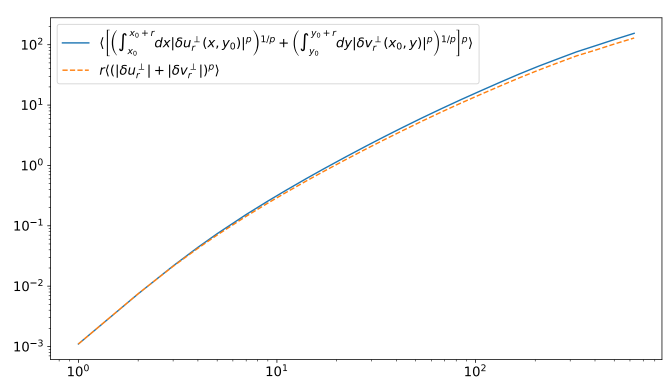

FIG. 1. Validation of the approximation performed between Eqs.(2) and (3) for $p = 2$, with $u = u_x$ and $v = u_y$. The angular brackets $\langle.\rangle$ indicate averaging in space.

[1] K. P. Iyer, K. R. Sreenivasan, and P. K. Yeung, Circulation in High Reynolds Number Isotropic Turbulence is a Bifractal, Physical Review X **9**, 041006 (2019).